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Chiral-soliton predictions for exotic baryons

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Abstract: We re-analyze the predictions of chiral-soliton models for the masses and decay widths of baryons in the exotic antidecuplet of flavour SU(3). The calculated ranges of the chiral-soliton moment of inertia and the π-nucleon scattering $\Sigma_{\pi N}$ term are used together with the observed baryon octet and decuplet mass splittings to estimate $1430\,\mathrm{MeV} < m_{\Theta^+} < 1660\,\mathrm{MeV}$ and $1790\ \mathrm{MeV} < m_{\Xi^{--}} < 1970\,\mathrm{MeV}$. These are consistent with the masses reported recently, but more precise predictions rely on ambiguous identifications of non-exotic baryon resonances. The overall decay rates of antidecuplet states are sensitive to the singlet axial-current matrix element in the nucleon. Taking this from polarized deep-inelastic scattering experiments, we find a suppression of the total Θ^+ and Ξ^{--} decay widths that may not be sufficient by itself to reproduce the narrow widths required by experiments. We calculate SU(3) breaking effects due to representation mixing and find that they tend to suppress the Θ^+ decay width, while enhancing that of the Ξ^{--}. We predict light masses for some exotic 27 baryons, including the $I = 1, J^P = \frac{3}{2}^+$ Θ^+ and $I = \frac{3}{2}, J^P = \frac{3}{2}^+$ Ξ multiplets, and calculate their decay widths.

Keywords: Solitons Monopoles and Instantons, 1/N Expansion, QCD, Chiral Lagrangians.

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Contents

1. Introduction

The constituent-quark model (CQM) has long reigned supreme as the default approach to hadron structure, masses and decays [1]. However, the CQM for light quarks has never been derived from QCD, and a complementary point of view is expressed in the chiral-soliton model (χSM) [2, 3]. Motivated by the observation that the short-distance current masses of the u, d and s quarks are all $\lesssim 100\,\mathrm{MeV}$, and the suggestion that chiral $\mathrm{SU}(3) \times \mathrm{SU}(3)$ may be a good symmetry of hadrons, the χSM treats baryons as topologically non-trivial configurations of the pseudoscalar meson fields. In some versions constituent quarks are considered to have been integrated out of the effective Lagrangian, whose solitons are interpreted as baryons. In the original form of the χSM proposed by Skyrme [4] the effective Lagrangian only contained terms up to quartic in the derivatives of the meson fields, a restriction we do not apply here. In other versions constituent quarks are explicitly present [2, 3], but the effective baryon theory has the same group-theoretical structure as purely mesonic models.

Despite considerable theoretical support and phenomenological exploration, the χSM has remained a minority interest, probably because it lacks the intuitive appeal and many of the phenomenological successes of the CQM. However, the χSM has its own successes, such as its prediction of 5/9 for the F/D ratio for axial-current nucleon matrix elements — which is arguably more successful than the CQM prediction of 2/3, the Guadagnini relation [5] between flavour SU(3) symmetry breaking in the lowest-lying baryon octet and decuplet, and the prediction that the singlet axial-current nucleon matrix element should be small [6].

The CQM and χSM are to a large degree complementary. Each of them reproduces certain aspects of hadronic physics and incorporates many features of QCD which are missing in the other. This complementarity between the CQM and χSM has some analogies to the relationship between the shell model and the droplet model of the atomic nucleus. Neither provides a complete description of the nucleus, but each one has its strengths. Faced with a new phenomenon, one should therefore try to understand it within each of the two approaches, and the best understanding may come from combining features of both approaches.

For a long time, a potential embarrassment for the χSM has been its prediction of exotic baryons. Beyond the lowest-lying $J = I = 1/2, 3/2$ baryons, the simple-minded SU(2) χSM predicted a tower of heavier $J = I = 5/2, 7/2, \ldots$ states, which have never been seen. However, the picture in the SU(3) version of the χSM is rather different: in this framework, the lowest-lying exotic baryon is an antidecuplet $\overline{10}$ [5],[7]–[10], with other exotic representations such as the $27, 35$ and $\overline{35}$ being heavier. For most of two decades, the existence of a light baryon antidecuplet has been a key unverified prediction of the χSM.

This exotic 'bug' may recently have turned into a feature, following the discovery of the exotic $\Theta^+(1540)$ baryon [11, 12] with a relatively low mass and small decay width as predicted in the χSM [8, 10, 13]. However, alternative postdictive interpretations of this state abound, including CQM descriptions [14]–[16], kaon-baryon molecules [17], kaon-Skyrmion bound states [18], etc. The χSM had also been used to predict the masses of the other baryons in the $\overline{10}$, including a Ξ^{--} with mass between 2070 MeV [8] and ~ 1850 MeV [10, 19], in strong correlation with the assumed value of the pion-nucleon sigma term. On the other hand the CQM approaches predicted $m_{\Xi^{--}} \lesssim 1760$ MeV [14, 15]. The NA49 collaboration has recently reported the observation of a candidate Ξ^{--} with a mass $\simeq 1860$ MeV [20], within this range of predictions. As we show below, a careful re-analysis of the results of Ref.[8] yields a range for the Ξ^{--} mass that includes the the experimental value.[1]

The purpose of this paper is to discuss critically the predictions for the exotic $\Theta(1540)^+$ and $\Xi_{\overline{10}}$ baryons within the χSM. As we discuss below, the masses and decays of the exotic $\overline{10}$ baryons are uniquely sensitive to the baryonic matrix elements of the SU(3)-singlet combinations of scalar $\bar{q}q$ densities and of axial $\bar{q}\gamma_\mu\gamma_5 q$ currents, respectively, and we discuss carefully the implications for exotic baryons of the experimental uncertainties in these quantities.

In order to predict the masses of the Θ^+ and Ξ^{--}, we use estimates of the chiral-invariant contributions from specific χSM calculations [10, 22], and estimates of the chiral $SU(3) \times SU(3)$ symmetry breaking contributions based the masses of octet and decuplet baryons and the $\Sigma_{\pi N}$ term in π-nucleon scattering [23]. Using the range 0.43 fm $< I_2 <$ 0.55 fm [10, 22] for the chiral-soliton moment of inertia that characterizes the difference between the $\overline{10}$ and 10 masses in the chiral limit, and the range 64 MeV $< \Sigma_{\pi N} <$ 79 MeV for the π-nucleon $\Sigma_{\pi N}$ term [23], we find the following ranges: 1430 MeV $< m_{\Theta^+} <$ 1660 MeV

[1]The relation of NA49 result to previous data is discussed in [21].

and 1790 MeV $< m_{\Xi^{--}} <$ 1970 MeV [2]. The more specific predictions made previously [8] relied on identifications of other resonances that are questionable, and/or a different value for the π-nucleon $\Sigma_{\pi N}$ term. We use values of χSM parameters inferred from the Θ^+ and Ξ^{--} masses to predict the masses of low-lying exotic baryons in a $J^P = \frac{3}{2}^+$ 27 representation of flavour SU(3) [19, 24], and calculate their decay widths.

In order to predict the decay rates of the Θ^+ and Ξ^{--}, one needs to know a specific combination of the octet and singlet axial-current matrix elements in the nucleon octet. In the absence of SU(3) symmetry breaking, in the leading order of the $1/N_c$ expansion, the χSM would predict that the singlet axial-current matrix element vanishes [6], in qualitative agreement with measurements of polarized deep-inelastic lepton-nucleon scattering. However, the deep-inelastic data indicate a small but non-zero singlet axial-current matrix element, which is accommodated by $1/N_c$ and $\mathcal{O}(m_s/\Lambda_{QCD})$ corrections in the χSM [25]. Inserting the value of the singlet axial-current matrix element extracted from polarized deep-inelastic lepton scattering experiments into the χSM formulae reduces somewhat the decay widths of the Θ^+ and Ξ^{--}, but perhaps not sufficiently to explain alone the very narrow widths of these states that are indicated by experiment [11, 12, 26]. Representation mixing introduces SU(3) breaking effects that suppress the Θ^+ decay width, while enhancing that of Ξ^{--}. They also have an important effect on the π-nucleon coupling that we calculate as well.

2. Review of relevant aspects of the χSM

We recall that the splittings between the centres of the lowest-lying octet, decuplet and antidecuplet baryons are given in the χSM by

$$\Delta M_{10-8} = \frac{3}{2I_1}\,, \qquad \Delta M_{\overline{10}-8} = \frac{N_c}{2I_2} = \frac{3}{2I_2}\,, \tag{2.1}$$

where $I_{1,2}$ are two soliton moments of inertia that depend on details of the chiral Lagrangian. Since $I_1,\ I_2 \sim \mathcal{O}(N_c)$, this means that $\Delta M_{\overline{10}-8} \sim \mathcal{O}(N_c^0)$, whereas ΔM_{10-8} is $\mathcal{O}(1/N_c)$. This has triggered some arguments [27] and counter-arguments [28], regarding the applicability of collective coordinate quantization to the $\overline{10}$. We note here that the application of the collective quantization relies on the rotor excitation being small in comparison with the classical mass. Since the latter is $\mathcal{O}(N_c)$, this requirement holds for $\overline{10}$ as well, even though the suppression is just $\mathcal{O}(1/N_c)$ vs. $\mathcal{O}(1/N_c^2)$ for the 10 and the 8. Experimentally, $\Delta M_{10-8} = 231$ MeV whereas $\Delta M_{\overline{10}-8} \sim 600$ MeV, in good agreement with formal N_c counting.[3]

The centre of the lightest octet of baryons is the average of the Λ and Σ masses, namely 1151.5 MeV, and the centre of the 10 of baryons is that of the Σ_{10}, namely 1382.1 MeV [29]. The centre of the $\overline{10}$ would likewise be identified with the $\Sigma_{\overline{10}}$, which may mix in general with the Σ_8 expected in the same band of soliton excitations, and even with other adjacent

[2]The considerably larger value of I_2 advocated in [22] would yield $\overline{10}$ masses that were unacceptably light, and specific model calculations correlate the values of I_2 and $\Sigma_{\pi N}$.

[3]Provided we interpret both the Θ^+ and the Ξ^{--} as members of the $\overline{10}$ multiplet.

Σ_8 states. Analogous mixing is expected for the $N_{\overline{10}}$. In the pioneering analysis of [8], the known $N(1710)$ was identified with the $N_{\overline{10}}$. However, such identifications are ambiguous, since the baryon spectrum is expected to contain both radial and rotational excitations that mix in general [9, 27, 28]. These identifications were abandoned in [30]. In this paper, we do not impose the identification of the $N(1710)$ or any other known nucleon resonance such as the $N(1440)$ with any combination of the solitonic $N_{\overline{10},8}$ states.

The leading-order chiral-symmetry breaking corrections to the lightest octet baryon masses are [8]:

$$N: +\frac{3}{10}\alpha + \beta - \frac{1}{20}\gamma\,, \tag{2.2}$$

$$\Lambda: +\frac{1}{10}\alpha + \frac{3}{20}\gamma\,, \tag{2.3}$$

$$\Sigma: -\frac{1}{10}\alpha - \frac{3}{20}\gamma\,, \tag{2.4}$$

$$\Xi: -\frac{1}{5}\alpha - \beta + \frac{1}{5}\gamma\,, \tag{2.5}$$

where the parameters α, β, γ cannot now be determined from first principles. In particular, β and γ are related to ratios of soliton moments of inertia [10, 22]:

$$\beta = -m_s \frac{K_2}{I_2}\,, \qquad \gamma = 2m_s\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right). \tag{2.6}$$

These origins impose on them some positivity conditions, namely:

$$\beta < 0\,, \qquad \frac{1}{2}\gamma - \beta > 0\,. \tag{2.7}$$

We also note that β and γ are formally of higher order in $1/N_c$ than α, and hence should be somewhat smaller than α. Whatever the values of α, β and γ, the octet baryons should obey the Gell-Mann-Okubo mass formula

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma\,, \tag{2.8}$$

which is quite well satisfied experimentally. In the case of the decuplet baryons, one has the leading-order mass corrections

$$\Delta: +\frac{1}{8}\alpha + \beta - \frac{5}{16}\gamma\,, \tag{2.9}$$

$$\Sigma^*: 0\,, \tag{2.10}$$

$$\Xi^*: -\frac{1}{8}\alpha - \beta + \frac{5}{16}\gamma\,, \tag{2.11}$$

$$\Omega: -\frac{1}{4}\alpha - 2\beta + \frac{5}{8}\gamma\,, \tag{2.12}$$

which provide the standard equal-spacing mass formula for the 10 multiplet:

$$m_{\Sigma^*} - m_\Delta = m_{\Xi^*} - m_{\Sigma^*} = m_\Omega - m_{\Xi^*}\,, \tag{2.13}$$

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which is also quite well satisfied.[4] As a bonus, one obtains the Guadagnini relation [5] between the 8 and 10 mass splittings:

$$8(m_{\Xi^*} + m_N) + 3m_\Sigma = 11m_\Lambda + 8m_{\Sigma^*}\,, \tag{2.14}$$

which is satisfied almost as accurately as (2.8) and (2.13). Finally, in the case of the $\overline{10}$ baryons, one has the mass corrections

$$\Theta^+ \,:\, +\frac{1}{4}\alpha + 2\beta - \frac{1}{8}\gamma\,, \tag{2.15}$$

$$N_{\overline{10}} \,:\, +\frac{1}{8}\alpha + \beta - \frac{1}{16}\gamma\,, \tag{2.16}$$

$$\Sigma_{\overline{10}} \,:\, 0\,, \tag{2.17}$$

$$\Xi_{I=3/2} \,:\, -\frac{1}{8}\alpha - \beta + \frac{1}{16}\gamma, \tag{2.18}$$

which also leads to equal spacings, but with magnitudes different from those in the decuplet of baryons.[5]

Reflecting the existence of the Guadagnini mass relation (2.14), we recall that the mass corrections (2) to (5) and (9) to (12) depend on just two combinations of the parameters α, β and γ, which may be determined as follows in a least-squares fit:

$$\alpha + \frac{3}{2}\gamma = -377\,\mathrm{MeV}\,, \tag{2.19}$$

$$\frac{1}{8}\alpha + \beta - \frac{5}{16}\gamma = -146\,\mathrm{MeV}\,. \tag{2.20}$$

A third relation is necessary if one is to determine α, β and γ and calculate the mass corrections (14) to (17). This can be provided by the chiral-symmetry breaking expression for the σ term in π-nucleon scattering:

$$\alpha + \beta = -\frac{2}{3}\frac{m_s}{m_u + m_d}\Sigma \tag{2.21}$$

where baryon and meson data yield the estimate $m_s/(m_u + m_d) = 12.9$ [32]. As is well-known, the value of $\Sigma_{\pi N}$ is related to the nucleon matrix element of the SU(3)-singlet combination $\langle N|(\bar{u}u + \bar{d}d + \bar{s}s)|N\rangle$.

3. Predictions for the masses of antidecuplet baryons

We have seen that, to predict the masses of the Θ^+ and Ξ^{--}, one must obtain values for the soliton moment of inertia I_2 and the chiral-symmetry breaking parameters α, β and γ. Different soliton models yield values for I_2 in the range [10, 22]:

$$0.43\,\mathrm{fm} < I_2 < 0.55\,\mathrm{fm}\,, \tag{3.1}$$

[4]We comment later on the potential significance of corrections of higher order in SU(3) symmetry breaking [31, 10].

[5]We note in passing that the CQM also predicts equal spacing for the $\overline{10}$ baryons, but different from that for the ordinary decuplet: $\Delta M_{\overline{10}} \sim (m_s - m_u)/3$, before the possible mixing of the $N_{8,\overline{10}}$ and $\Sigma_{8,\overline{10}}$.

which yields the range

$$538\,\mathrm{MeV} < \Delta M_{\overline{10}-8} < 638\,\mathrm{MeV}\,. \tag{3.2}$$

In the version of the Skyrme model discussed in [2] the upper limit on I_2 is even higher, being of the order of 1 fm. We note, however, that such a value of I_2 would bring the $\overline{10}$ masses unacceptably low.

To determine the chiral-symmetry breaking corrections, we use the central values of two recent determinations of the π-nucleon Σ term: $\Sigma_{\pi N} = 64 \pm 8$ (79 ± 7) MeV [23]. We now have three equations for the three unknowns α, β and γ, for which we find the values needed to predict $\overline{10}$ baryon masses in the χSM:

$\Sigma_{\pi N}$	64 ± 8 MeV	79 ± 7 MeV
α	-489 ± 103 MeV	-683 ± 90 MeV
β	-61 ± 34 MeV	3 ± 30 MeV
γ	74 ± 69 MeV	203 ± 60 MeV

(3.3)

Using the ranges (3.2), (3.3), we find the following ranges for the masses of the exotic baryons in the $\overline{10}$ multiplet:

$$\text{for } \Sigma_{\pi N} = 64\,\mathrm{MeV}: \quad m_{\Theta^+} = 1505^{+84}_{-66}\,\mathrm{MeV}\,, \quad m_{\Xi^{--}} = 1885^{+84}_{-66}\,\mathrm{MeV} \tag{3.4}$$

$$\text{for } \Sigma_{\pi N} = 79\,\mathrm{MeV}: \quad m_{\Theta^+} = 1569^{+84}_{-66}\,\mathrm{MeV}\,, \quad m_{\Xi^{--}} = 1853^{+84}_{-66}\,\mathrm{MeV}\,, \tag{3.5}$$

where the upper and lower errors reflect the variation of I_2, eq. (3.1). An additional error comes from the $\sim$7 MeV uncertainty in the central values of $\Sigma_{\pi N}$: $\delta m_{\Theta^+}/\delta\Sigma_{\pi N} \approx 4$, $\delta m_{\Xi^{--}}/\delta\Sigma_{\pi N} \approx 2$. Overall, we find the ranges

$$1432\,\mathrm{MeV} < m_{\Theta^+} < 1657\,\mathrm{MeV}\,, \qquad 1786\,\mathrm{MeV} < m_{\Xi^{--}} < 1970\,\mathrm{MeV}\,, \tag{3.6}$$

upon combining these errors in quadrature.

The ranges (3.6) certainly include the observed masses m_{Θ^+}=1539$\pm$2 MeV and $m_{\Xi^{--}}$ $= 1862 \pm 2$ MeV, but more precise predictions cannot be made without introducing more assumptions. In our view, the success of the prediction of [8] for the Θ^+ mass was somewhat fortuitous. Ref. [8] identified the $N_{\overline{10}}$ with the $N(1710)$, and assumed an older value for the π-nucleon Σ term: $\Sigma_{\pi N} = 45$ MeV. It is this latter value, in particular, that was responsible for the unsuccessful prediction in [8] of a very heavy mass ~ 2070 MeV for the Ξ^{--} state. This conclusion is reinforced by the analysis of ref. [33] where $\Sigma_{\pi N} = 74 \pm 12$ MeV is obtained from the observed spectrum of usual and exotic baryons.

The ratio of $m_s/(m_u + m_d) = 12.9$ that we have assumed in the above analysis corresponds to the strange quark mass $m_s = 140$ MeV for $m_u + m_d = 11$ MeV. It is, however, quite possible that quark masses in the effective models take values different than in the underlying QCD theory. In [22] the best fit value of the strange quark mass was approximately 185–195 MeV, rather than 140 MeV. Since m_s and $\Sigma_{\pi N}$ enter as a product into (2.21), one can compensate the large value of the latter by increasing m_s. This would introduce another 25% uncertainty into the estimates (3.6). In what follows, we do not use the value of the $\Sigma_{\pi N}$ term any more, but fit the model parameters to the measured baryon masses.

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Within the χSM framework, the observed masses of the Θ^+ and Ξ^{--} can be used, together with the masses of ordinary octet and decuplet, to estimate the key model parameters, whose interpretation we discuss below:

$$I_1 = 1.27 \text{ fm}\,, \qquad I_2 = 0.49 \text{ fm}\,, \qquad \alpha = -605\,\text{MeV}, \beta = -23\,\text{MeV}\,, \qquad \gamma = 152\,\text{MeV}\,, \tag{3.7}$$

corresponding to $\Sigma_{\pi N} = 73\,\text{MeV}$. We see again that the reported mass of the $\Xi_{\overline{10}}$ is no problem for the χSM. Having fixed all the parameters of the model we predict the remaining $\overline{10}$ masses: $M_{N^*} = 1646\,\text{MeV}$ and $M_\Sigma = 1754\,\text{MeV}$.

We now check the consistency of the leading-order expansion in SU(3) symmetry breaking, by incorporating the representation mixing due to the SU(3)-breaking Hamiltonian:

$$\hat{H}' = \alpha D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} D^{(8)}_{8i} \hat{S}_i\,, \tag{3.8}$$

Most relevant for this paper are the following mixings induced by (3.8):

$$\begin{aligned} |B_8\rangle &= |8_{1/2}, B\rangle + c^B_{\overline{10}} |\overline{10}_{1/2}, B\rangle + c^B_{27} |27_{1/2}, B\rangle\,, \\ |B_{10}\rangle &= |10_{3/2}, B\rangle + a^B_{27} |27_{3/2}, B\rangle + a^B_{35} |35_{3/2}, B\rangle\,, \\ |B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d^B_8 |8_{1/2}, B\rangle + d^B_{27} |27_{1/2}, B\rangle + d^B_{\overline{35}} |\overline{35}_{1/2}, B\rangle \end{aligned} \tag{3.9}$$

where

$$\begin{gathered} c^B_{\overline{10}} = c_{\overline{10}} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, c^B_{27} = c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \\ \sqrt{6} \end{bmatrix}, a^B_{27} = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, a^B_{35} = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix} \\ d^B_8 = d_8 \begin{bmatrix} 0 \\ \sqrt{5} \\ \sqrt{5} \\ 0 \end{bmatrix}, \qquad d^B_{27} = d_{27} \begin{bmatrix} 0 \\ \sqrt{3/10} \\ 2/\sqrt{5} \\ \sqrt{3/2} \end{bmatrix}, \qquad d^B_{\overline{35}} = d_{\overline{35}} \begin{bmatrix} 1/\sqrt{7} \\ 3/(2\sqrt{14}) \\ 1/\sqrt{7} \\ \sqrt{5/56} \end{bmatrix} \end{gathered} \tag{3.10}$$

in the basis $[N, \Lambda, \Sigma, \Xi]$, $[\Delta, \Sigma^*, \Xi^*, \Omega]$ and $[\Theta^+, N_{\overline{10}}, \Sigma_{\overline{10}}, \Xi_{\overline{10}}]$ respectively, and

$$\begin{aligned} c_{\overline{10}} &= -\frac{I_2}{15}\left(\alpha + \frac{1}{2}\gamma\right), \qquad c_{27} = -\frac{I_2}{25}\left(\alpha - \frac{1}{6}\gamma\right), \\ a_{27} &= -\frac{I_2}{8}\left(\alpha + \frac{5}{6}\gamma\right), \qquad a_{35} = -\frac{I_2}{24}\left(\alpha - \frac{1}{2}\gamma\right), \\ d_8 &= \frac{I_2}{15}\left(\alpha + \frac{1}{2}\gamma\right), \qquad d_{27} = -\frac{I_2}{8}\left(\alpha - \frac{7}{6}\gamma\right), \qquad d_{\overline{35}} = -\frac{I_2}{4}\left(\alpha + \frac{1}{6}\gamma\right). \end{aligned} \tag{3.11}$$

For our set of parameters (3.7) the mixing coefficients range from 0.06 to 0.36:

$$\begin{aligned} c_{\overline{10}} &= -d_8 = 0.088\,, \qquad & c_{27} &= 0.063\,, \\ a_{27} &= 0.150\,, & a_{35} &= 0.071\,, \\ d_{27} &= 0.245\,, & d_{\overline{35}} &= 0.362 \end{aligned} \tag{3.12}$$

which by (3.10) results in admixtures which are typically of the order of 10 to 20%.

These first-order admixtures lead to the following second-order corrections to the masses of 8, 10 and $\overline{10}$ baryons [10]:

$$\begin{aligned}
E_8^{(2)}(Y,T) &= -I_2\Big[\frac{1}{60}\left(Y+\left(T(T+1)-\frac{1}{4}Y^2\right)+\frac{1}{2}Y^2\right)\left(\alpha+\frac{1}{2}\gamma\right)^2+ \\
&\quad +\frac{1}{250}\left(9-\frac{5}{2}\left(T(T+1)-\frac{1}{4}Y^2\right)-\frac{7}{4}Y^2\right)\left(\alpha-\frac{1}{6}\gamma\right)^2\Big] \\
&= -I_2\Big[\frac{1}{60}\left(Y+T\,(T+1)+\frac{1}{4}Y^2\right)\left(\alpha+\frac{1}{2}\gamma\right)^2+ \\
&\quad +\frac{1}{250}\left(9-\frac{5}{2}T\,(T+1)-\frac{9}{8}Y^2\right)\left(\alpha-\frac{1}{6}\gamma\right)^2\Big], \\
E_{10}^{(2)}(Y) &= -I_2\left[\frac{1}{16}\left(1+\frac{3}{4}Y+\frac{1}{8}Y^2\right)\left(\alpha+\frac{5}{6}\gamma\right)^2+\frac{5}{336}\left(1-\frac{1}{4}Y-\frac{1}{8}Y^2\right)\left(\alpha-\frac{1}{2}\gamma\right)^2\right], \\
E_{\overline{10}}^{(2)}(Y) &= I_2\Big[\frac{1}{30}\left(1+\frac{1}{2}Y-\frac{1}{2}Y^2\right)\left(\alpha+\frac{1}{2}\gamma\right)^2-\frac{1}{640}\left(8-6Y+Y^2\right)\left(\alpha-\frac{7}{6}\gamma\right)^2- \\
&\quad -\frac{3}{896}\left(8+2Y-Y^2\right)\left(\alpha+\frac{1}{6}\gamma\right)^2\Big]. \qquad (3.13)
\end{aligned}$$

We find, in particular, the following dominant second-order corrections to the Θ^+ and $\Xi_{\overline{10}}$ masses due to mixing with other exotic rotational excitations:

$$\begin{aligned}
\delta_2 m_{\Theta^+} &= -\frac{3}{112}I_2\left(\alpha+\frac{1}{6}\gamma\right)^2, \\
\delta_2 m_{\Xi^{--}} &= -I_2\left(\frac{3}{128}\left(\alpha-\frac{7}{6}\gamma\right)^2+\frac{15}{896}\left(\alpha+\frac{1}{6}\gamma\right)^2\right), \qquad (3.14)
\end{aligned}$$

where the effect comes from mixing with similar states in a 27 multiplet (related to $(\alpha - 7\gamma/6)$) and a $\overline{35}$ multiplet (related to $(\alpha+\gamma/6)$). Using the values of α and γ extracted above (3.7), these corrections amount numerically to -22.5, $-50\,\mathrm{MeV}$, respectively.

It is likely that there are similar mass corrections due to mixing with other states such as radial excitations [27]. Some of these mixings have been considered in [9] in a specific model, but there could be additional effects of this type which have not yet been fully investigated in the literature.[6] If included in the above fit to the exotic baryon masses, the corrections (3.14) would correspond to shifting $I_2 \to 0.51$ fm and $\Sigma_{\pi N} \to 72\,\mathrm{MeV}$. These small changes indicate that the procedure [8] of calculating mass corrections to first order in $1/N_c$ and SU(3) symmetry breaking may be reasonably stable.

It is reassuring to note that the extracted values of α and β correspond to a value of $\Sigma_{\pi N}$ between the two recent experimental determinations [23]. We also note that the extracted values of β and γ (3.7) respect the positivity constraints (2.7) required in the χSM, and that $|\alpha| \gg |\beta|, |\gamma|$, as expected on the basis of the $1/N_c$ expansion. Inserting the value $\beta = -23\,\mathrm{MeV}$ (3.7), extracted from the masses of the known antidecuplet states, the χSM expression for β (2.6) and the estimate $m_s \sim 100 - 200\,\mathrm{MeV}$ suggest the following

[6] Such mixing is likely to be more important for non-exotic baryons, which is one reason why we do not advocate estimating I_2 from fits to baryon masses including quadratic corrections.

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value for the ratio of two moments of inertia:

$$\frac{K_2}{I_2} = 0.23 - 0.11\,, \tag{3.15}$$

which is quite small. However, a realistic error on β might be 35 MeV, in which case somewhat larger values of K_2 would also be possible. We note also that (2.1) and the observed octet and decuplet masses yield $I_1 = 1.29$ fm. Considering now the χSM expression for γ (2.6), we see that the small ratio K_2/I_2 (3.15) is quite consistent with the positive value of γ found in (3.7), and yields the following value for the ratio of two other moments of inertia in the χSM:

$$\frac{K_1}{I_1} = 0.98 - 0.49 \tag{3.16}$$

for $m_s = 100 - 200$ MeV. The extracted values of the four moments of inertia $I_{1,2}$ and $K_{1,2}$ provide interesting constraints on the χSM that lie beyond the scope of this paper, though we note that they are not typical of model calculations.

4. Predictions for the decay widths of exotic baryons

4.1 General Remarks on Decay Widths in the χSM

Whilst the mass spectra discussed in the previous Section are given as systematic expansions in both N_c and m_s in a theoretically controllable way, reliable predictions for the decay widths cannot be organized in a similar manner. As explained below, they depend on modelling and 'educated' guesses, and hence are subject to additional uncertainties.

The width for any decay $B \to B' + \varphi$ may be expressed in terms of the matrix element $\mathcal{M}$ and a two-body phase-space factor:

$$\Gamma_{B\to B'\varphi} = \frac{\overline{\mathcal{M}^2}}{8\pi M\, M'} p_\varphi\,, \tag{4.1}$$

where φ is a pseudoscalar meson with momentum p_φ in the B rest frame:

$$p_\varphi = \frac{\sqrt{(M^2 - (M' - m_\varphi)^2)(M^2 - (M' - m_\varphi)^2)}}{2M}$$

and the bar over $\mathcal{M}^2$ in (4.1) denotes an average over the initial and a sum over the final spins, and — when explicitly indicated — summing and averaging over isospin.

The first uncertainty comes from the fact that the baryon masses M and M' appear in the denominator of (4.1) yielding, formally, *infinite series* in N_c and m_s. The same holds for the momentum of the outgoing meson φ. It is a common practice to treat the phase factor exactly, rather than expand it up to a given order in N_c and m_s, despite the fact that in the matrix element $\mathcal{M}$ only a few first terms in $1/N_c$ and m_s/Λ_{QCD} are calculated.

Secondly, $\mathcal{M}$ stands in (4.1) for the *relativistic* matrix element which, in the case of nucleon decay, could be calculated from the Lagrangian density considered already by Adkins, Nappi and Witten in Ref. [4]:

$$\mathcal{L}_{\rm int} = i g_{\pi NN}\, \pi^a \left(\overline{\psi}\gamma_5 \tau_a \psi\right). \tag{4.2}$$

Unfortunately, we have at our disposal a *non-relativistic* model, in which baryons are considered as infinitely heavy, rather than a relativistic field theory like (4.2). It was already observed in [4] that the non-relativistic reduction of (4.2) leads to the interaction Lagrangian (as extended to SU(3)):

$$L_{\rm int} = g\, \partial^i \varphi^\alpha \, A_{\alpha i} \,, \tag{4.3}$$

where $A_{i\alpha}$ is a spatial component of an axial current of flavour α. Here, g is a coupling constant related to $g_{\pi NN}$ which depends, in principle, on the initial and final baryon states [8]. Furthermore, it is clear that the Lagrangian density for spin-3/2 baryons decaying into baryons of spin 1/2 cannot be cast in the form (4.2), because it must involve a Rarita-Schwinger spinor which carries an extra vector index and has different canonical dimension. Luckily, even in this case, one can still use (4.3), but the coupling constant g should be appropriately rescaled [8].

It is appropriate at this point to keep in mind the well-known and difficult problem of Yukawa couplings in χSM(see e.g. [35] for an in-depth discussion). The fundamental source of this problem is that baryons are constructed from meson fields and so in leading order in $1/N_c$ terms linear in mesons vanish when one expands around the soliton configuration.

There have been several interesting attempts to resolve this problem, but at present there is no consensus about their effectiveness Since we are focusing here on exotics, a detailed discussion of this problem would take us much beyond the scope of the present paper.

The baryon decay operator following from (4.3) can be written as

$$\hat{O}^{(8)}_\varphi = 3\left[G_0 D^{(8)}_{\varphi i} - G_1 d_{ibc}\, D^{(8)}_{\varphi b}\hat{S}_c - G_2 \frac{1}{\sqrt{3}} D^{(8)}_{\varphi 8}\hat{S}_i\right] p_{\varphi\, \iota} \,, \tag{4.4}$$

which transforms as an octet of SU(3). The decay matrix elements $\mathcal{M} = \langle B'|\hat{O}^{(8)}_\varphi|B\rangle$ may then be written in terms of the couplings $G_{0,1,2}$, which are in turn related to axial-current matrix elements $a_{0,1,2}$:

$$A_{\alpha i} = a_0 D_{\alpha i} - a_1 d_{ibc}\, D^{(8)}_{\alpha b}\hat{S}_c - \frac{a_2}{\sqrt{3}} D^{(8)}_{\alpha 8}\hat{S}_i \tag{4.5}$$

by generalized Goldberger-Treiman relations:

$$G_i = g a_i \,. \tag{4.6}$$

As discussed in [34], there are m_s and $1/N_c$ corrections to such relations associated with form factors in axial-current matrix elements. Their calculation would involve a treatment of deformations of soliton configurations, of which the principles and one example are given in [34], but which have never been calculated in a model-independent way. Here we use (4.6) and comment later on the possible impact of deviations from it.

In what follows, we first make the approximation, following (4.3), that g is a *universal* constant and calculate the decay widths using (4.1), which is a good initial approximation, since decuplet and antidecuplet decays are governed by two different linear combinations of the coupling constants $G_{0,1,2}$, and, even using the constraints from the semileptonic hyperon decays, one has enough freedom to accommodate simultaneously large decuplet widths and small $\overline{10}$ widths. However, when we also include the leading m_s corrections, or try to estimate the $g_{\pi NN}$ coupling, or the suppressed decay widths of the $\overline{10}$ baryons and the widths of more exotic states such as those in the 27 multiplets, then we need to know $G_{0,1,2}$ *separately.* Then it becomes important whether the corrections due to the mass dependence of g are included or not. Here, we include them following [8], and discuss the potential uncertainties in our predictions that they reflect.

At leading order in the $1/N_c$ expansion, corresponding to ultra-non-relativistic baryons, all the couplings of the $8, 10$ and $\overline{10}$ baryons to pseudoscalar mesons are proportional to the dimensionless constant G_0 introduced above, with $F/D = 5/9$ [36]. In this approximation, the chiral soliton has no coupling to the singlet pseudoscalar-meson field, and the singlet axial-current matrix elements vanish. This provides a qualitative explanation [6] for the smallness of the singlet axial-current matrix element of the nucleon inferred from polarized deep-inelastic lepton scattering experiments.

The constants $G_{1,2}$ are non-leading as far as N_c counting is concerned. However, in antidecuplet decays the G_1 contribution gets an additional N_c enhancement from the SU(3)-flavour Clebsch-Gordan coefficients calculated in large N_c limit [37].

One source of m_s corrections is representation mixing. As already discussed in connection with baryon masses, in the presence of SU(3) breaking the physical states are no longer pure octet, decuplet or antidecuplet states, but contain admixtures (3.9) of the order of m_s. Since their magnitudes are completely determined by the mass splittings, their influence on the decay widths can be estimated reliably. In the following, we use them below as estimates of the possible errors in the decay widths associated with SU(3) symmetry breaking.

In addition to these calculable effects, the operator $\hat{O}^{(8)}_{\varphi}$ gets additional m_s corrections whose algebraic structure is known from the analysis of semileptonic hyperon decays [38]. These introduce three additional couplings, which we ignore in the present phenomenological analysis, as their determinations would require a lengthy analysis together with hyperon decays, which lies beyond the scope of this paper.

4.2 SU(3) symmetry limit

Neglecting mixing between baryon multiplets, one has [25]

$$g_{\pi NN} = \frac{7}{10}\left(G_0 + \frac{1}{2}G_1 + \frac{1}{14}G_2\right), \tag{4.7}$$

$$G_{10} = G_0 + \frac{1}{2}G_1\,, \tag{4.8}$$

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2 \tag{4.9}$$

which are related to $g_{\pi\Delta N}$ and $g_{K\Theta N}$, $g_{\pi\Xi N}$ respectively. Furthermore

$$\frac{F}{D} = \frac{5}{9}\left(\frac{G_0 + \frac{1}{2}G_1 + \frac{1}{2}G_2}{G_0 + \frac{1}{2}G_1 - \frac{1}{6}G_2}\right). \tag{4.10}$$

The G_2 coupling is related à la Goldberger-Treiman to the singlet axial-current matrix element in the nucleon:

$$G_2 = \frac{2m_N}{3F_\pi} g_A^0 \,, \tag{4.11}$$

for $F_\pi = 93\,\mathrm{MeV}$ and may be non-zero. However, the consistency of the $1/N_c$ expansion would require G_2 to be relatively small, along with G_1.

To proceed further, we need input from Δ decay. Using (4.8), the measured decuplet decay width $\Gamma_\Delta = 115 \div 125\,\mathrm{MeV}$ [29] and the theoretical prediction for the Δ width

$$\Gamma_\Delta = \frac{3G_{10}^2}{8\pi M_\Delta M_N} \frac{1}{5} p_\pi^3$$

would yield the combination

$$G_{10} = G_0 + \frac{1}{2} G_1 = 22.4 \,. \tag{4.12}$$

Unfortunately, we have no independent experimental information on any other combination of G_0 and G_1. Luckily, for non-exotic matrix elements, we only need G_{10} and G_2. Therefore one finds

$$g_{\pi NN} = \frac{7}{10} G_{10} + \frac{G_2}{20} = 15.6 + \frac{G_2}{20} \,, \tag{4.13}$$

where the G_2-dependent correction is presumably small, in view of (4.11). The value (4.13) does not compare well with the experimental range $g_{\pi NN} = 13.3 \pm 0.1$ given in [39] or the slightly different range $g_{\pi NN} = 13.13 \pm 0.07$ recently advocated in [40], and is not useful for extracting a numerical value of the undetermined parameter G_2 wanted for calculating the decay widths of the Θ^+ and Ξ^{--}. Likewise, the baryon F/D ratio is not known sufficiently well to extract a useful value of G_2. However, we recall that the longitudinal asymmetry in polarized deep-inelastic lepton-nucleon scattering is sensitive to the nucleon singlet axial-current matrix element, and use the value of g_A^0 extracted from these experiments to estimate G_2:

$$g_A^0 = 0.3 \pm 0.1 \to G_2 \simeq 2 \,, \tag{4.14}$$

which is indeed small compared with G_{10}.

In order to predict $G_{\overline{10}}$ it is necessary to know a new combination of G_0 and G_1. For this, we first seek guidance from χSM calculations, which yield [25, 8]:

$$G_1 = (0.5 \pm 0.1) \times G_0 \,. \tag{4.15}$$

Using the central value in (4.12), we would find

$$G_0 \simeq 17.9, \qquad G_1 \simeq 8.9 \,. \tag{4.16}$$

Inserting (4.14) and (4.16) into (4.10), we find $F/D \simeq 0.59$, which should be compared with the experimental value 0.56 ± 0.02. Moreover, the value (4.14) worsens only slightly the leading-order prediction (4.13) for $g_{\pi NN}$.

Inserting the values (4.16), (4.14) into the expression (4.9), we find $G_{\overline{10}} = 7.9$, which is considerably smaller than either $g_{\pi NN}$ or $g_{\pi \Delta N}$. However, this suppression is insufficient to explain fully the narrow widths of the Θ^+ and Ξ^{--}, as suggested in [8]. For example, the total width of the Θ^+, which decays into KN, would be given by

$$\Gamma_{\Theta^+} = \frac{3G_{\overline{10}}^2}{8\pi M_{\Theta^+} M_N} \frac{1}{5} p_K^3 = 20.6\,\mathrm{MeV}\,. \tag{4.17}$$

Although this number is relatively small, it is considerably larger than recent experimental estimates [12]. For comparison, we recall that the Θ^+ decay width is formally of higher order in $1/N_c$ than that of the Δ [37], and that the CQM would suggest that $G_1/G_0 = 4/5, G_2/G_0 = 2/5$, which would predict a strong suppression of Γ_{Θ^+} and $\Gamma_{\Xi^{--}}$.

In view of the mixed success of the above calculation of baryon couplings in the χSM, we explore the corrections due to the initial assumption of universality in the coupling g entering (4.3).

It was argued in [8] that the theoretical predictions for the decay widths should be multiplied by the ratio M'/M, however their numerical values are consistent [9] with multiplying decuplet decays by an inverse ratio[7] M/M' and antidecuplet decays by M'/M. It is beyond the scope of the present paper to discuss the origin of these corrections, here we try to examine various approximations present in the literature, and this is just one of the more important ones. The results below are displayed in terms of multiplicative factors so it is easy to 'undo' them, if the reader would like to understand the impacts of different assumptions.

It is convenient to split these factors into N_c-dependent and m_s independent corrections that are identical for the whole multiplet, and additional m_s corrections that have to be calculated for each single decay separately:

$$\frac{M}{M'} = \frac{M_{10}}{M_8}\left(\frac{M/M_{10}}{M'/M_8}\right) = 1.2 \times \left(\frac{M/M_{10}}{M'/M_8}\right) \equiv 1.2 \times R^{(g)}_{B\to B'}\,, \tag{4.18}$$

where we have evaluated the N_c dependent correction using $M_{10} = 1382.1\,\mathrm{MeV}$ and $M_8 = 1151.5\,\mathrm{MeV}$ for the mean decuplet and octet masses, respectively, and $R^{(g)}_{B\to B'}$ is an m_s-dependent correction, which for the $\Delta \to N + \pi$ transition amounts to 1.09. We see that inclusion of the factor M_{10}/M_8 reduces the value of G_{10} by $\sqrt{1.2} = 1.0954$, to

$$G_{10} = 20.4 \tag{4.19}$$

resulting in

$$g_{\pi NN} = 14.3 + \frac{G_2}{20}, \quad G_{\overline{10}} = 7.2\,. \tag{4.20}$$

The width of Θ^+ has to be modified now by the ratio $M_8/M_{\overline{10}} = 0.66$ (for $M_{\overline{10}} = 1754\,\mathrm{MeV}$), yielding

$$\Gamma_{\Theta^+} = \frac{3G_{\overline{10}}^2}{8\pi M_{\Theta^+} M_N} \frac{M_8}{M_{\overline{10}}} \frac{1}{5} p_K^3 = 11.1\,\mathrm{MeV} \tag{4.21}$$

which agrees with the original prediction of [8]. The excellent agreement in the second case is mainly due to the suppression factor $M_8/M_{\overline{10}}$.

[7] The authors [41] claim there was a misprint in [8], where the ratio for the decuplet decays was inadvertently written as M'/M. Different opinions are presented in [9, 42].

One may, alternatively, invert the logic and use the measured Θ^+ width, which is presumably smaller than 10 MeV, to extract independently values of G_0 and G_1. For this, we consider two extreme cases: $\Gamma_{\Theta^+} = 10$ and 1 MeV. Then, without the $M_8/M_{\overline{10}}$ correction we would obtain

$$\begin{aligned} G_{\overline{10}} &= 5.5 \quad \text{for} \quad \Gamma_{\Theta^+} = 10\,\text{MeV}\,, \\ G_{\overline{10}} &= 1.75 \quad \text{for} \quad \Gamma_{\Theta^+} = 1\,\text{MeV}\,, \end{aligned} \tag{4.22}$$

where we have chosen the positive sign in order to keep $G_0 > G_1$. Assuming $G_2 = 2$, we get two sets of solutions

$$\begin{aligned} G_0 &= 17.1\,, \quad G_1 = 10.6 \quad \text{for } \Gamma_{\Theta^+} = 10\,\text{MeV}\,, \\ G_0 &= 15.8\,, \quad G_1 = 13.1 \quad \text{for } \Gamma_{\Theta^+} = 1\,\text{MeV}\,. \end{aligned} \tag{4.23}$$

The corresponding ratios G_1/G_0 lie somewhat outside the model ranges quoted previously, but are still below unity. We see that the freedom stemming from the fact that non-exotic decays fix only one linear combination of G_0 and G_1 enables one to accommodate a very narrow width of the Θ^+ without changing the prediction for Δ.

We can now repeat the same for the expressions corrected by the multiplet mass ratios, and obtain

$$\begin{aligned} G_{\overline{10}} &= 6.82 \quad \text{for} \quad \Gamma_{\Theta^+} = 10\,\text{MeV}\,, \\ G_{\overline{10}} &= 2.16 \quad \text{for} \quad \Gamma_{\Theta^+} = 1\,\text{MeV}\,. \end{aligned} \tag{4.24}$$

Using $G_{10} = 20.25$, we get

$$\begin{aligned} G_0 &= 16.1, \quad G_1 = 8.30 \quad \text{for} \quad \Gamma_{\Theta^+} = 10\,\text{MeV}\,, \\ G_0 &= 14.6, \quad G_1 = 11.4 \quad \text{for} \quad \Gamma_{\Theta^+} = 1\,\text{MeV}\,. \end{aligned} \tag{4.25}$$

Note that here the ratios G_1/G_0 are closer to the model range.

We remark that, irrespective of whether we correct the widths by the $M_R/M_{R'}$ ratio, or not,

$$\begin{aligned} \Gamma_{\Xi^{--}_{\overline{10}} \to \Xi^- + \pi^-} &\sim 1.3\Gamma_{\Theta^+} \\ \Gamma_{\Xi^{--}_{\overline{10}} \to \Sigma^- + K^-} &\sim 0.8\Gamma_{\Theta^+}\,. \end{aligned} \tag{4.26}$$

Hence, we predict total decay widths for Θ and Ξ^{--} which are similar to within a factor of 2. As we show below, this relation is removed when we include symmetry-breaking terms. Numerically the width of $\Xi^{--}_{\overline{10}}$, which was recently estimated by the NA49 Collaboration to be below 18 MeV becomes 21 and 2.1 MeV for the two extreme cases ($\Gamma_{\Theta^+} = 10$ and 1 MeV) discussed above.[8]

For convenience in the subsequent analysis, we adopt the following parametrization of the χSM model couplings:

$$G_1 \equiv \rho\, G_0\,, \qquad G_2 \equiv \epsilon G_0 = \left(\frac{9(F/D) - 5}{3(F/D) + 5}\right)(\rho + 2)G_0\,, \tag{4.27}$$

[8]Another recent theoretical estimate puts the $\Xi_{\overline{10}}$ width ≤ 10 MeV [43].

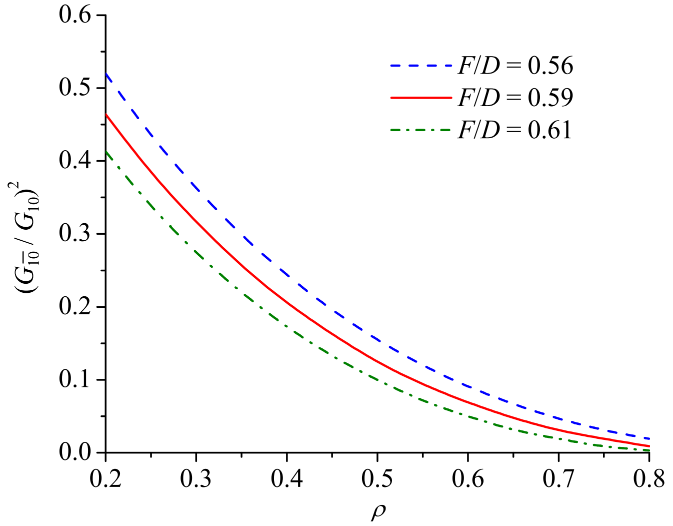

Figure 1: Values of the ratios $(G_{\overline{10}}/G_{10})^2$ for different values of F/D, as functions of the parameter $\rho = G_1/G_0$.

where the last equation follows from (4.10). The various fits described above yield $\rho \lesssim 0.8$, as compared to the favoured model range $\rho = 0.5 \pm 0.1$ (4.15).

In order to check the sensitivity of the suppression mechanism for $\overline{10}$ decays to the numerical values of couplings G_i, we plot in figure 1 the ratios $(G_{\overline{10}}/G_{10})^2$ for different values of F/D as functions of the parameter ρ. We see that $\overline{10}$ decays are suppressed with respect to 10 for a wide range of ρ. Further suppression, if one follows the logic of [8], may be provided, as discussed above, by the rescaling of the decay widths by the average multiplet mass ratios, and by the m_s corrections, as discussed in the following section. In the numerical evaluations below, we vary ρ from 0.2 to 0.8 while fixing $F/D = 0.59$.

4.3 SU(3) Symmetry Breaking Effects

We now calculate the SU(3)-breaking corrections due to the baryon representation mixing and the non-universality of the g coupling discussed above, as an aid to assessing the SU(3)-breaking uncertainties in the analysis of section 4.2. There is some ambiguity in the way these corrections are treated. As already discussed, by using experimental values for the masses M and M' in (4.1), as well as for p_φ, we *implicitly* sum an infinite series of m_s/Λ_{QCD} corrections. In the following, however, we compute the square of $\mathcal{M}$ up to terms linear in m_s stemming from the representation mixing due to the mass splitting hamiltonian H'. We recall that there are also corrections to the decay operator $O_\varphi^{(8)}$, which – as explained above – are ignored in the following. We shall also include residual m_s corrections coming from the ratios $(M/M_{10})/(M'/M_8)$ for decuplet decays and $(M'/M_8)/(M/M_{\overline{10}})$ for antidecuplet decays, respectively.

Decuplet baryons can decay only to octet baryons, and we have

$$\begin{aligned}\left\langle B_8'\right| \hat{O}_{\varphi}^{(8)} \left|B_{10}\right\rangle = & \left\langle 8_{1/2}, B'\right| \hat{O}_{\varphi}^{(8)} \left|10_{3/2}, B\right\rangle + \\ & + a_{27}^{B} \left\langle 8_{1/2}, B'\right| \hat{O}_{\varphi}^{(8)} \left|27_{3/2}, B\right\rangle + c_{27}^{B'} \left\langle 27_{1/2}, B'\right| \hat{O}_{\varphi}^{(8)} \left|10_{3/2}, B\right\rangle ,\end{aligned} \tag{4.28}$$

where we include mixings with the 27 multiplets of baryons, which were neglected in [8, 28]. We then introduce [44]

$$G_{10} \equiv G_0 + \frac{1}{2} G_1 , \qquad G_{27} \equiv G_0 - \frac{1}{2} G_1 , \qquad G_{27}' \equiv G_0 - 2 G_1 . \tag{4.29}$$

In terms of these combinations, the baryon representation mixing discussed earlier yields [44]

$$\overline{\mathcal{M}_{\Delta}^2} = \frac{3}{5} G_{10} \left[G_{10} + \frac{10}{3} a_{27}\, G_{27} + \frac{2}{3} c_{27}\, G_{27}' \right] p_{\varphi}^2 \tag{4.30}$$

for the squared matrix element to first order in m_s/Λ_{QCD}.

Using the previous numbers for the mixing coefficients, the ratio of the new expression for Δ decay to the old one is

$$R_{\Delta\to N}^{(\mathrm{mix})} = 1 + \frac{0.499\, G_{27} + 0.042\, G_{27}^{'}}{G_{10}}$$

It can be seen from figure 2 that $R_{\Delta\to N}^{(\mathrm{mix})}$ is quite insensitive to the value of parameter ρ. For the specific value $\rho = 1/2$, we find

$$R_{\Delta\to N} = R_{\Delta\to N}^{(g)} R_{\Delta\to N}^{(\mathrm{mix})} = 1.09 \times 1.3 = 1.42 \tag{4.31}$$

in which case the value of G_{10} extracted in (4.19) should be reduced by a factor 1/1.19, to

$$G_{10} = 17.1 \tag{4.32}$$

when we include the SU(3) breaking due to representation mixing. On the other hand, if we neglect factors of mass ratios from (4.18), then we should reduce the coupling of (4.12) by a factor 1/1.14, to

$$G_{10} = 19.65 . \tag{4.33}$$

In either case, the G_{10} coupling is reduced and, as we see below, the prediction for $g_{\pi NN}$ is improved.

Finally, we calculate the corresponding SU(3) corrections to the π-nucleon coupling constant due to representation mixing, which can be obtained from the formula

$$g_{\pi NN} = \left| \langle p_8 | \hat{O}_{\pi^0}^{(8)} | p_8 \rangle \right| \frac{1}{p_{\pi}} , \tag{4.34}$$

where we work in the frame $\vec{p}_{\pi} = (0, 0, p_{\pi})$. This gives [44]

$$g_{\pi NN} = \frac{7}{10} \left[G_0 + \frac{1}{2} G_1 + \frac{1}{14} G_2 \right] + c_{\overline{10}} G_{\overline{10}} + \frac{2}{15} c_{27} H_{27} , \tag{4.35}$$

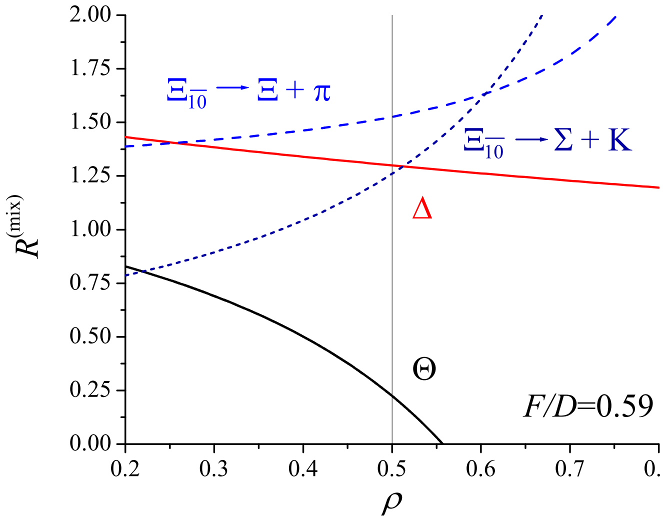

Figure 2: The correction factors $R^{(\mathrm{mix})}$ due to SU(3)-breaking representation mixing for the decays discussed in the text, as functions of the parameter $\rho \equiv G_1/G_0$.

where we have introduced

$$G_{\overline{10}} \equiv G_0 - G_1 - \frac{1}{2}G_2\,, \qquad H_{27} \equiv G_0 - 2G_1 + \frac{3}{2}G_2\,. \tag{4.36}$$

The expression (4.35) may be written in the form

$$g_{\pi NN} = G_{10} \times \left(\frac{0.796 + 0.245\rho + 0.019\epsilon}{1 + 0.5\rho}\right), \tag{4.37}$$

which yields $g_{\pi NN} = 13.2$ to 12.2 for $G_{10} = 17.1$, as found in (4.32) after including representation mixing, and $\rho = 0.2$ to 0.8. This result obtained including representation mixing compares better with the experimental range $g_{\pi NN} = 13.3 \pm 0.1$ [39] or 13.13 ± 0.07 [40] than did the leading-order prediction (4.13), and leaves open the possibility that a more complete calculation of m_s/Λ_{QCD} and $\mathcal{O}(1/N_c)$ effects - including those discussed in [34] — might remove the discrepancy completely. We plot in figure 3 the π-nucleon coupling $g_{\pi NN}$ as a function of the parameter $\rho \equiv G_1/G_0$ for $F/D = 0.59$ and four different values of G_{10} discussed in the text (4.12), (4.19), (4.32), (4.33). In general, this example warns us that, to the accuracy they are currently made, χSM calculations of couplings are subject to uncertainties of $\mathcal{O}(20)\%$.

In the absence of SU(3)-symmetry breaking, antidecuplet baryons can decay directly only to octet baryons. Including first-order m_s/Λ_{QCD} effects, we find:

$$\begin{aligned}\langle B_8'|\,\hat{O}^{(8)}_{\varphi}\,|B_{\overline{10}}\rangle = {} & \left\langle 8_{1/2}, B'\right|\hat{O}^{(8)}_{\varphi}\left|\overline{10}_{1/2}, B\right\rangle + \\ & + d_8^B \left\langle 8_{1/2}, B'\right|\hat{O}^{(8)}_{\varphi}\left|8_{1/2}, B\right\rangle + d_{27}^B \left\langle 8_{1/2}, B'\right|\hat{O}^{(8)}_{\varphi}\left|27_{1/2}, B\right\rangle + \\ & + c_{\overline{10}}^{B'} \left\langle \overline{10}_{1/2}, B'\right|\hat{O}^{(8)}_{\varphi}\left|\overline{10}_{1/2}, B\right\rangle + c_{27}^{B'} \left\langle 27_{1/2}, B'\right|\hat{O}^{(8)}_{\varphi}\left|\overline{10}_{1/2}, B\right\rangle\,,\end{aligned} \tag{4.38}$$

where only the term proportional to $c_{\overline{10}}^{B'}$ and d_{27}^B were taken into account previously [8, 28]. In the presence of SU(3) breaking, decays of antidecuplet baryons into decuplet baryons

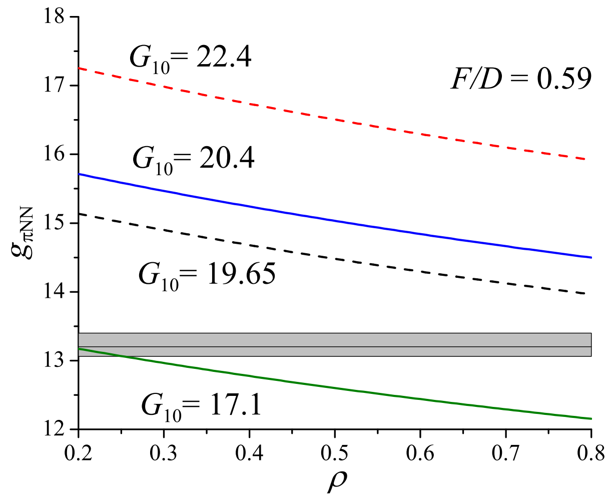

Figure 3: The π-nucleon coupling $g_{\pi NN}$ as a function of the parameter $\rho \equiv G_1/G_0$ for $F/D = 0.59$ and four different values of G_{10} discussed in the text (4.12), (4.19), (4.32), (4.33). The upper shaded band corresponds to the experimental evaluation of [39], and the lower shaded band corresponds to the range in [40].

also become possible, via a matrix element

$$\langle B'_{10} | \hat{O}^{(8)}_{\pi^-} | B_{\overline{10}} \rangle = d^B_{27} \langle 10_{3/2}, B' | \hat{O}^{(8)}_{\varphi} | 27_{1/2}, B \rangle + a^{B'}_{27} \langle 27_{3/2}, B' | \hat{O}^{(8)}_{\varphi} | \overline{10}_{1/2}, B \rangle \tag{4.39}$$

whose magnitude we discuss later.

For the discussion of these decays, we introduce the following constants [44]

$$\begin{aligned} H_{\overline{10}} &\equiv G_0 - \frac{5}{2} G_1 + \frac{1}{2} G_2 \,, \qquad & H'_{\overline{10}} &\equiv G_0 + \frac{11}{14} G_1 + \frac{3}{14} G_2 \,, \\ H_8 &\equiv G_0 + \frac{1}{2} G_1 - \frac{1}{2} G_2 \,, \qquad & H'_8 &\equiv G_0 + \frac{1}{2} G_1 - \frac{1}{6} G_2 \,. \end{aligned} \tag{4.40}$$

In terms of these, we find for the average of the $\Theta^+ \to p + K^0$ and $\Theta^+ \to n + K^+$ decays:

$$\overline{\mathcal{M}^2}_{\Theta^+ \to N+K} = \frac{3}{10} G_{\overline{10}} \left[G_{\overline{10}} + \frac{5}{2} c_{\overline{10}} H_{\overline{10}} - \frac{7}{2} c_{27} H'_{\overline{10}} \right] \times p^2 \,. \tag{4.41}$$

This formula resembles that given in [8], but there are some differences:

- The squared decay matrix element is not just a function of $G_{\overline{10}}$. In fact, when $|G_{\overline{10}}|$ is comparable to the SU(3)-breaking corrections, one should use the full quadratic expression for $\overline{\mathcal{M}^2}_{\Theta^+ \to N+K}$, rather than the linear form (4.41). Even if $G_{\overline{10}} = 0$, Θ^+ could still decay through the mixing terms, contrary to the impression given by [8, eq. (56)] (see however [43]).

- There is an additional term due to mixing with the 27 representation, which is not small and was not included in [8].

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- The squaring of the matrix element introduces a factor of 2 which is not apparent in [8]. This has been corrected in [43].

The corresponding squared amplitude for the $\Xi^{--}_{\overline{10}} \to \Xi^- + \pi^-$ decay reported by NA49 has the form:

$$\overline{\mathcal{M}^2}_{\Xi^{--}_{\overline{10}} \to \Xi^- + \pi^-} = \frac{3}{10} G_{\overline{10}} \left[G_{\overline{10}} + \frac{7}{3} c_{27}\, H'_{\overline{10}} + \frac{2}{3} d_{27} H_{27} \right] \times p^2 \,. \tag{4.42}$$

Another possible decay of this state is $\Xi^{--}_{\overline{10}} \to \Sigma^- + K^-$, whose squared matrix element is given by

$$\overline{\mathcal{M}^2}_{\Xi^{--}_{\overline{10}} \to \Sigma^- + K^-} = \frac{3}{10} G_{\overline{10}} \left[G_{\overline{10}} - \frac{5}{2} c_{\overline{10}}\, H_{\overline{10}} + \frac{7}{6} c_{27}\, H'_{\overline{10}} - \frac{2}{3} d_{27}\, H_{27} \right] \times p^2. \tag{4.43}$$

These examples exhibit explicitly that the SU(3)-breaking corrections to $\overline{10}$ decays are *not* universal.

Let us define the correction factor coming from the representation mixing:

$$R^{(\mathrm{mix})}_{\Theta^+ \to N + K} = 1 + \frac{0.22\ H_{\overline{10}} - 0.22\ H'_{\overline{10}}}{G_{\overline{10}}}. \tag{4.44}$$

In figure 2 we plot $R^{(\mathrm{mix})}_{\Theta^+ \to N+K}$ as a function of parameter ρ. It can be seen that it is rather sensitive to value of ρ, yielding for $\rho = 1/2$

$$R^{(\mathrm{mix})}_{\Theta^+ \to N + K} = 0.2 \,. \tag{4.45}$$

The correction from the non-universality of the g coupling is

$$R^{(g)}_{\Theta^+ \to N + K} = \frac{M_N / M_8}{M_{\Theta^+} / M_{\overline{10}}} = 0.93 \,. \tag{4.46}$$

These two corrections act in a similar way, tending to suppress the decay rate of Θ^+ by a further factor of ~ 0.25, reinforcing the χSM prediction that the Θ^+ should be very narrow, and emphasizing that the SU(3)-breaking corrections are potentially very significant in this case.

In the case of the Ξ decays, we have

$$\begin{aligned} R^{(\mathrm{mix})}_{\Xi^{--} \to \Xi^- + \pi^-} &= 1 + \frac{0.15\ H'_{\overline{10}} + 0.16\ H_{27}}{G_{\overline{10}}}, \\ R^{(\mathrm{mix})}_{\Xi^{--} \to \Sigma^- + K^-} &= 1 + \frac{-0.22\ H_{\overline{10}} + 0.07\ H'_{\overline{10}} - 0.16\ H_{27}}{G_{\overline{10}}}. \end{aligned} \tag{4.47}$$

We see from figure 2 that $R^{(\mathrm{mix})}_{\Xi^{--} \to \Xi^- + \pi^-}$ is a slowly-varying function of ρ, while $R^{(\mathrm{mix})}_{\Xi^{--} \to \Sigma^- + K^-}$ is close to 1 in the vicinity of $\rho = 1/2$. Numerically, for $\rho = 1/2$ we obtain:

$$\begin{aligned} R^{(\mathrm{mix})}_{\Xi^{--} \to \Xi^- + \pi^-} &= 1.535 \,, \\ R^{(\mathrm{mix})}_{\Xi^{--} \to \Sigma^- + K^-} &= 1.269 \,. \end{aligned} \tag{4.48}$$

The g correction reads in this case

$$R^{(g)}_{\Xi^{--}\to\Xi^-+\pi^-} = \frac{M_\Xi/M_8}{M_{\Xi_{\overline{10}}}/M_{\overline{10}}} = 1.08\,,$$
$$R^{(g)}_{\Xi^{--}\to\Sigma^-+K^-} = \frac{M_\Sigma/M_8}{M_{\Xi_{\overline{10}}}/M_{\overline{10}}} = 0.98\,. \tag{4.49}$$

Despite small suppression in the last case, the m_s corrections tend to increase the width of $\Xi_{\overline{10}}$, reinforcing the message that the corrections to $\overline{10}$ decays are not universal.

Finally, we consider the decay $\Xi^-_{\overline{10}} \to \Xi^{*0} + \pi^-$, preliminary evidence for which was recently mentioned by NA49 [45]. Since this decay is not allowed in the SU(3) symmetry limit, it can only go via admixtures of 27 multiplets in the $\overline{10}$ and/or 10, as given in (4.39). Calculating the relevant matrix element, we get [44]

$$\overline{\mathcal{M}^2}_{\Xi^-_{\overline{10}}\to\Xi^{*0}+\pi^-} = \frac{1}{162}\left[d_{27}\left(G_0 - 2G_1\right) + a_{27}\left(G_0 + G_1\right)\right]^2 p^2\,. \tag{4.50}$$

This matrix element is extremely small, approximately two orders of magnitudes smaller than the one for Θ^+ decay[9] (4.41). Furthermore, the masses in the denominator of (4.1) give another factor of 1/2, yielding the decay rate

$$\Gamma_{\Xi^-_{\overline{10}}\to\Xi^{*0}+\pi^-} \sim \left(\frac{1}{200} \div \frac{1}{100}\right)\Gamma_{\Theta^+}\,. \tag{4.51}$$

Therefore this mixing mechanism is unlikely to be the explanation of the preliminary evidence reported by NA49.

Within the CQM, an interpretation of this decay as due to the decay of an isodoublet Ξ state within an octet of pentaquarks, which is degenerate with the Ξ in the $\overline{10}$, was recently proposed [46]. There is no additional rotational octet excitation in the χSM, and it was therefore argued in [46] that the confirmation of this decay would be a challenge for the χSM. However, we remark that octets are expected as vibrational excitations in the χSM, but with properties that are very difficult to estimate. Nevertheless, $1/N_c$ arguments suggest that these vibrational excitations should have masses comparable to the exotic rotational excitations discussed above. An alternative explanation of the $\Xi^-_{\overline{10}} \to \Xi^{*0} + \pi^-$ decay, offered in the next Section, is that the Ξ^- state reportedly observed is a member of the $(27, \frac{3}{2})$ that might be almost degenerate with that in the $\overline{10}$.

Let us briefly summarize the findings of this Section. First, we have shown that the m_s/Λ corrections are not universal. Secondly, they are rather large and in some cases, such as the Θ^+ decay rate, sensitive to the parameter $\rho = G_1/G_0$ which we have varied between 0.2 and 0.8. One should not be surprised that these corrections are large, since the leading term is small and vanishes exactly in the quark model limit of the χSM,[10], whereas no other matrix element vanishes in this limit. This means that some antidecuplet decays may be controlled primarily by representation mixing. Thirdly, we have calculated the decay width of $\Xi_{3/2}$ to $\Xi^*(1530)$ which can only go through the admixture of 27 and found out that it was 2 orders of magnitude smaller than the width of Θ^+.

[9]Note that the meson momenta p are identical for both decays, to within 2 MeV.

[10]Strictly speaking, the enhancement factors $R^{(\mathrm{mix})}$ would diverge in this limit and lose physical meaning.

5. Predictions for the masses of other exotic baryons

As already mentioned, the SU(3) χSM predicts a tower of heavier and more exotic baryons, of which the lightest is expected to be a 27 representation with $J^P = \frac{3}{2}^+$. Several numerical estimates have been made for the masses of the exotic Θ_1, Σ_2 and Ω_1 baryons in these multiplets [19, 24], where the symbols specify the strangeness (hypercharge) and the subscripts specify the isospins of these states. In light of the previous analysis, using the masses of the Θ^+ and $\Xi_{\overline{10}}$ as inputs, we now refine these predictions.

We recall that the splittings between the centres of the lowest-lying 27-plet, octet and decuplet baryons are given in the χSM by

$$\Delta M_{(27,\frac{3}{2})-(10,\frac{3}{2})} = \frac{1}{I_2}\,, \qquad \Delta M_{(27,\frac{1}{2})-(8,\frac{1}{2})} = \frac{5}{2I_2}\,, \tag{5.1}$$

the chiral-symmetry breaking mass corrections within the $(27, \frac{3}{2})$ multiplet are

$$\Theta_1 : +\frac{1}{7}\alpha + 2\beta - \frac{5}{14}\gamma\,, \tag{5.2}$$

$$\Sigma_2 : +\frac{5}{56}\alpha - \frac{25}{112}\gamma\,, \tag{5.3}$$

$$\Omega_1 : -\frac{13}{56}\alpha - 2\beta + \frac{65}{112}\gamma\,, \tag{5.4}$$

where the subscript denotes the isospin of a given baryon in the 27-plet. Using the values of I_2, α, β and γ extracted previously (3.7) from the observed Θ^+ and $\Xi_{\overline{10}}$ masses, we estimate for the exotic baryons in the $(27, \frac{3}{2})$ multiplet:

$$\left(27, \frac{3}{2}\right) : \quad \begin{aligned} m_{\Theta_1} &= 1597\,\mathrm{MeV}\,, & m_{\Sigma_2} &= 1695\,\mathrm{MeV}\,, \\ m_{\Xi_{3/2}} &= 1876\,\mathrm{MeV}\,, & m_{\Omega_1} &= 2057\,\mathrm{MeV}\,. \end{aligned} \tag{5.5}$$

We note that the $\Xi_{3/2}$ in the $(27, \frac{3}{2})$ is almost degenerate with $\Xi_{3/2}$ in $\overline{10}$. As discussed in the previous section, this might be relevant to the preliminary evidence of a state at 1860 MeV decaying into $\Xi(1530)^0 + \pi$ [45]. Such a decay is not allowed for the $\Xi_{3/2}$ in the $\overline{10}$, since $\overline{10} \notin 10 \times 8$, but it would be allowed for a $\Xi_{3/2}$ in the 27, since $27 \in 10 \times 8$.

The spectra of the exotic baryons found at first order in SU(3) symmetry breaking in the $\overline{10}$ and $(27, \frac{3}{2}^+)$ representations are shown in figure 4.

It should be emphasized that the $1/N_c$ expansion used in the χSM approach becomes less reliable for heavier baryons, so these numerical predictions should be treated as only approximate. However, we confirm previous suggestions [19, 24] that there may be an isospin triplet of $S = -1$ Θ_1 baryons weighing barely 60 MeV more than the Θ^+, and the presence of a low-lying I=2 Σ multiplet is also suggested. These would both have $J^P = \frac{3}{2}^+$, with the corresponding $(27, \frac{1}{2}^+)$ being significantly heavier. If found, these exotic 27 baryons would provide further encouragement for the χSM approach.

For comparison, a recent detailed study [47] of exotic baryon spectroscopy in the CQM suggests the existence of a $(\overline{10}, \frac{3}{2}^+)$ excitation of the Θ^+ with a mass within about 100 MeV of the Θ^+ (see also [48]), a slightly heavier Θ_1 state in the $(27, \frac{1}{2}^+)$ and a rather heavier

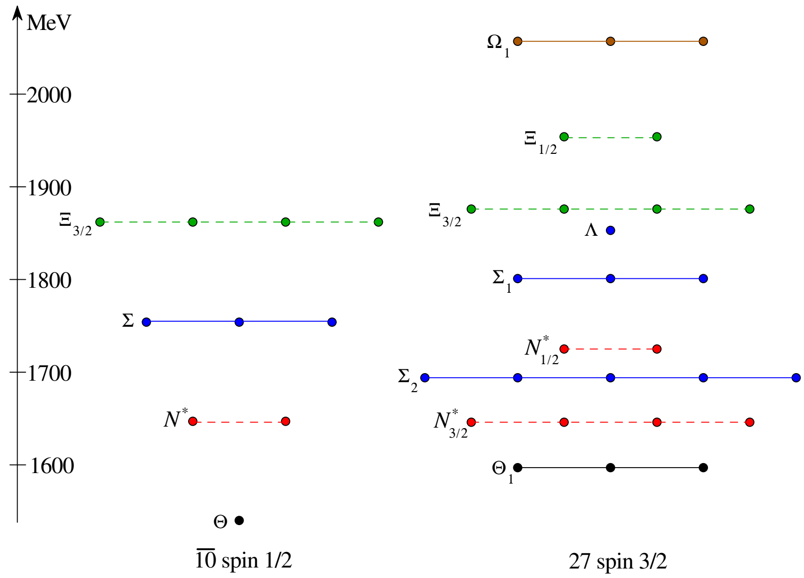

Figure 4: The spectra of exotic baryons found at first order in SU(3) symmetry breaking, using parameters fitted from the Θ^+ and $\Xi_{\overline{10}}$ masses. The $(\overline{10}, \frac{1}{2}^+)$ spectrum is shown on the left, and the $(27, \frac{3}{2}^+)$ spectrum on the right.

Θ_1 state in the $(27, \frac{3}{2}^+)$. In this approach, the exotic baryons with $Y < 2$ are significantly lighter than in our χSM estimates above: in particular, the Ξ state in the $(\overline{10}, \frac{1}{2}^+)$ is considerably lighter than was recently reported [20].

It is interesting to exhibit explicitly the mass difference of the lightest members of the $(\overline{10}, \frac{1}{2})$ and $(27, \frac{3}{2})$ multiplets:

$$\Delta_\Theta = M_{\Theta_{27}} - M_{\Theta^+_{\overline{10}}} = \frac{1}{2}\left(3\frac{1}{I_1} - \frac{1}{I_2}\right) - \frac{1}{56}\left(13\gamma + 6\alpha\right). \tag{5.6}$$

We see that, for the set of parameters (3.7), partial cancellations occur in each bracket, yielding $\Delta_\Theta = 63\,\mathrm{MeV}$. The lowest isospin triplet in the $(27, \frac{3}{2})$ multiplet is only slightly heavier than the Θ^+. A similar cancellation occurs for Ξ states:

$$\Delta_\Xi = M_{\Xi_{27}} - M_{\Xi_{\overline{10}}} = \frac{1}{2}\left(3\frac{1}{I_1} - \frac{1}{I_2}\right) + \frac{1}{112}\left(13\gamma + 6\alpha\right) \tag{5.7}$$

which yields $\Delta_\Xi = 18.7\,\mathrm{MeV}$ for the set of parameters (3.7).

Although this looks like an accidental cancellation, it is actually quite robust, and would persist even if we did not assume that the mass of the $\Xi_{\overline{10}}$ is 1860 MeV. This is illustrated in figure 5, where we plot the $\overline{10}$ spectrum, together with the $(27, \frac{3}{2})$ states Θ_1 and $\Xi_{3/2}$ (dashed lines) as functions of the π-nucleon sigma term $\Sigma_{\pi N}$. In making this plot, we have taken as inputs only the masses of the non-exotic states and of the Θ^+, in order to determine α, β, γ and I_2, but have not used the mass of the $\Xi_{\overline{10}}$. We see that the lowest $(27, \frac{3}{2})$ state Θ_1 is only a few tens of MeV above Θ^+, and that the Ξ states are almost degenerate, for a large range of $\Sigma_{\pi N}$.

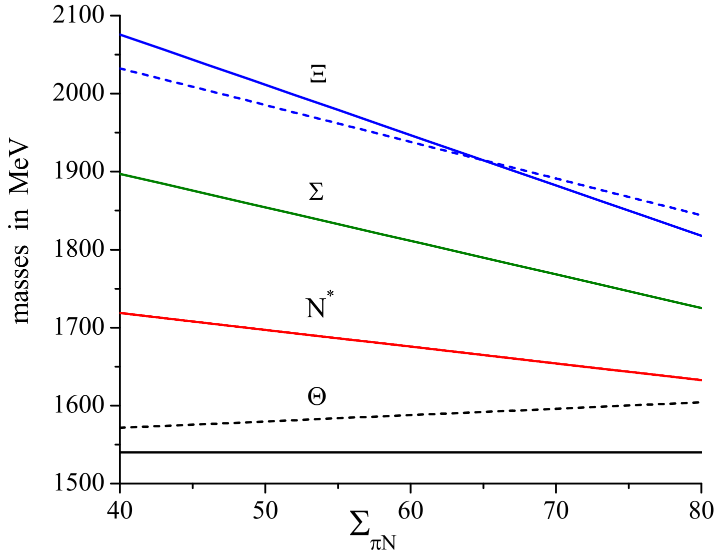

Figure 5: The spectra of $(\overline{10}, \frac{1}{2})$ baryons (solid lines) together with the masses of the Θ_1 and $\Xi_{3/2}$ in the $(27, \frac{3}{2})$ (dashed lines) as functions of $\Sigma_{\pi N}$, using parameters fitted from the masses of the Θ^+ and non-exotic states.

One should therefore consider the possibility that NA49 has already seen the Ξ_{27} state decaying to $\Xi^*(1530)$. In order to test this hypothesis, let us calculate the decay width:

$$\Gamma_{B_{27}\to B'_{10}\varphi} = \frac{1}{8\pi}\frac{F_{27}^2}{MM'}\frac{25}{72}\left(\begin{array}{cc|c} 8 & 10 & 27 \\ \varphi & B' & B \end{array}\right)^2 p_\varphi^3 \,, \tag{5.8}$$

where [44]

$$F_{27} = G_0 - \frac{1}{2}G_1 - \frac{3}{2}G_2 \,. \tag{5.9}$$

Let us note that, similarly to $G_{\overline{10}}$, F_{27} vanishes in the CQM limit (i.e., for $G_1/G_0 = 4/5$ and $G_2/G_0 = 2/5$), and so we expect the decay width (5.8) to be small. Indeed, for the values of $G_{1,2}$ given in (4.23), (4.25) we get $F_{27} = 6$ to 9, which is still bigger than $G_{\overline{10}}$ but smaller than G_{10}. Moreover, for the decay $\Xi^-(1560) \to \Xi^*(1530) + \pi^0$ there is another suppression factor, namely the square of the SU(3) Clebsch-Gordan coefficient entering (5.8), which is 1/6. Altogether the width is of the order of 1 MeV:

$$\Gamma_{\Xi_{27}^- \to \Xi_{10}^{*0} + \pi^-} \sim 0.6 \div 1.5\,\mathrm{MeV}\,, \tag{5.10}$$

depending on the value of F_{27} and the mass of Ξ_{27}^-.

For $27_{3/2} \to 8_{1/2}$ we obtain (not summed or averaged over isospin):

$$\Gamma_{B_{27}\to B'_{8}\varphi} = \frac{1}{8\pi}\frac{G_{27}^2}{MM'}\frac{4}{9}\left(\begin{array}{cc|c} 8 & 8 & 27 \\ \varphi & B' & B \end{array}\right)^2 p_\varphi^3 \tag{5.11}$$

where

$$G_{27} = G_0 - \frac{1}{2}G_1 = G_{10} - G_1. \tag{5.12}$$

For the values of $G_{1,2}$ given in (4.23), (4.25) we get $G_{27} = 9$ to 12, yielding rather large 27-plet widths. In the case of the Θ_{27}^{++}, the Clebsch-Gordan coefficient in (5.11) is unity, and we get

$$\Gamma_{\Theta_{27}^{++} \to p+K^+} \sim 37 \div 66\,\mathrm{MeV}\,. \tag{5.13}$$

This is rather larger than the width of the $\Theta_{\overline{10}}^{+}$. Moreover, since this is a decay from spin 3/2 to spin 1/2, the logic of [8] would imply a non-universality factor M/M' that would increase the 27 widths even further. Searches for $I = 1$ 'partners' of the $\Theta_{\overline{10}}^{+}$ need to take this into account, together with the negative results of previous experimental searches [49].

Similarly, we obtain $41 \div 77$ MeV for the total width of $\Xi(27, \frac{3}{2})^-$, implying a very small branching ratio $\lesssim 0.02$ for the decay into $\Xi(1530) + \pi^-$, shown in (5.10). This poses two challenges for the interpretation of the preliminary NA49 data [45] as decay of a $\Xi^-(27, \frac{3}{2})$: one is that the total production rate of the $\Xi^-(27, \frac{3}{2})$ would need to be larger by a factor 50 or so to compensate for the small branching ratio, and the other is that the natural width would probably exceed the NA49 limit.

6. Summary

We have examined carefully the predictions of the χSM for the masses and widths of the exotic baryons Θ^+ and $\Xi_{\overline{10}}$. It was a non-trivial success for the χSM to have predicted the existence of such relatively light exotic states [8], candidate members of a novel $\overline{10}$ multiplet of baryons [7]. The old complaint that the χSM predicts unobserved exotic particles has been refuted. The CQM did not predict such states, although it may accommodate them. A key untested prediction of the χSM is that the Θ^+ and $\Xi_{\overline{10}}$ should have $J^P = \frac{1}{2}^+$. Some versions of the CQM suggest instead $J^P = \frac{1}{2}^-$, but $J^P = \frac{1}{2}^+$ can be accommodated in variants of the CQM with strongly-bound diquarks [14, 15].

Dynamical calculations of soliton moments of inertia [10, 22] and a realistic assessment of our knowledge of chiral symmetry breaking contributions to baryon masses [23] could have been used to predict ranges for their masses that include the observed values, but with uncertainties ~ 200 MeV. The remarkable prediction of [8], although somewhat fortuitous, exhibits an important feature of the soliton models, namely the fact that exotic states are much lighter than naive expectations of the quark model, which would predict the lightest strange pentaquark to weight of the order of 1700 MeV. This χSM is an inevitable consequence of the requirement that the second-order m_s corrections do not spoil the non-exotic spectra, and that the π-nucleon $\Sigma_{\pi N}$ term lies within the modern phenomenological range.

There is almost no doubt today that the lightest member of the exotic antidecuplet has been discovered. We have used its mass and the latest determinations of the π-nucleon $\Sigma_{\pi N}$ term [23] to predict successfully the mass of the $\Xi_{\overline{10}}$, as also done in [10] and [19]. These predictions, however, rely on the determination of the $\Sigma_{\pi N}$ term which have been varying over the last 20 years between 45 and 77 MeV. Nevertheless, the very existence of exotic $\Xi_{\overline{10}}$ and Ξ^+ states between 1830 and 2000 GeV is an unavoidable prediction of the chiral soliton models, as can be seen in figure 5.[11]

Quark models have been modified [14, 15] to accommodate light pentaquarks by introducing quark correlations, otherwise absent in the naive formulations. The positive parity of the new exotic states, which is an unproved key prediction of the soliton models, has been accommodated as well. However, some versions of the quark models [50] as well as lattice calculations [51] and QCD sum rules [52] predict negative parity. Therefore, the measurement of the parities and spins of exotic baryons is one of the most important experimental challenges. It is, however, a tall order, especially if one realizes that the parity and spin of the Ω^-, whose discovery was a milestone in the foundations of our present understanding of the strong interactions, have still not been measured until today [29].

The recent announcement of NA49 of the discovery of some members of an exotic Ξ multiplet with masses around 1860 MeV would constitute, if confirmed [21], another success of the soliton model. As we have already discussed above, the model is quite flexible in accommodating a Ξ mass in the wide range between 1830 to 2000 GeV. However, it is encouraging that the mass reported by NA49 and recent estimate of the π-nucleon $\Sigma_{\pi N}$ term [23] are consistent within the model accuracy [33]. On the other hand, predictions of the $\Xi_{\overline{10}}$ masses in the correlated CQM lie below 1800 MeV [14, 15], possibly indicating the need for additional degrees of freedom.

One of the most striking predictions of χSM calculations was the successful prediction of a narrow decay width for the Θ^+ [8]. Other calculations predicted a larger decay width [9], partly because they lacked the G_2 term which is however small, partly because the model calculations of the remaining $G_{1,2}$ constants gave a smaller cancellation than the phenomenological fit of [8], and partly because the larger Θ^+ mass was used enhancing the phase space factor p^3. We find that the Θ^+ decay width is suppressed for values of the χSM couplings that lie close to the ranges favoured in models, and that it is further suppressed by the SU(3)-breaking effects due to representation mixing. In comparison, the CQM has available some suitable dynamical suppression mechanisms based on colour and spatial overlap arguments [53] and selection rules [54]. Another possible suppression mechanism has been recently proposed within the framework of the CQM, involving mixing between the two nearly degenerate states that arise in models with two diquarks and an antiquark [55].

The narrowness of the Θ^+ in chiral soliton models is far from being intuitive. It occurs due to the cancellation of the couplings in the collective decay operator as a conspiracy of the SU(3) group-theoretical factors and phenomenological values of these couplings. This cancellation is, however, by no means accidental. Indeed, in the small soliton limit the cancellation is exact. If in the χSM one artificially sets the soliton size $r_0 \to 0$, then the model reduces to free valence quarks which, however, 'remember' the soliton structure [56]. In this limit, many quantities are given as ratios of group-theoretical factors, yielding

[11] These numbers do not include the uncertainties of about 50 MeV due to $\mathcal{O}(m_s^2)$ corrections.

famous quark model results: $g_A = 5/3$, $\Delta\Sigma = 1$ and $\mu_p/\mu_n = -2/3$. Therefore the small-soliton limit is a very useful theoretical tool for understanding the predictions of soliton models.

In order to get reliable estimates of the individual couplings, rather than only of the combinations which enter in the decuplet and antidecuplet decay widths separately, we have discussed various corrections. Following [8], we have multiplied the widths by the appropriate mass ratios and also by the correction factors due to representation mixing. These factors are found to be large, so model predictions for the decay widths suffer from large uncertainties. Incidentally, these corrections tend coherently to suppress the width of Θ^+, while the width of $\Xi_{\overline{10}}$ is coherently enhanced.

Are there any exotics beyond the $\overline{10}$? In soliton models one gets a tower of exotic states starting with $(27, \frac{3}{2})$, $(35, \frac{5}{2})$, etc. Whether they can easily be seen is another issue. As one can see from figure 5, the existence of a relatively light isotriplet of Θ_1 states belonging to the $(27, \frac{3}{2})$ representation, just a few tens of MeV above the Θ^+, is quite a robust prediction of the soliton models. Unlike the antidecuplet Θ^+ though, the decay widths of $(27, \frac{3}{2})$ states to ordinary octet baryons are relatively large. We have estimated $\Gamma_{\Theta_{27}^{++}} \sim 37 \div 66\,\mathrm{MeV}$, with a possible enhancement due to the correction factors discussed in the text. Furthermore, 27 baryons, unlike the $\overline{10}$ ones, can decay into ordinary decuplet baryons. However, the widths of these decays are small and comparable to the decay widths of $\overline{10}$ to 8. Again in this case the effective decay coupling vanishes in the small soliton limit discussed above.

Interestingly, another quite robust prediction of the present model is the existence of the nearly degenerate $I = 3/2$ Ξ multiplets in the $\overline{10}$ and $(27, \frac{3}{2})$ representations. The decay $\Xi(1860)^- \to \Xi^{*0} + \pi^-$ recently reported by NA49 could be interpreted as an observation of Ξ_{27}. However, all charged states of Ξ_{27} must be found in order to confirm this hypothesis. Moreover, the rather large total width of the Ξ_{27} obtained in the present work poses serious challenges for such an interpretation. On the other hand, in the correlated CQM such a decay is naturally explained [46] as as the decay of the Ξ isodoublet belonging to a nearly degenerate pentaquark octet.

Therefore, the observation of $\Xi(1860)^{--}$ and $\Xi(1860)^+$ decays into decuplet would suggest discovery of yet another tower of exotic states. However, the non-observation of these decays, together with positive evidence for $\Xi(1860)^-$ and $\Xi(1860)^0$ decays to decuplet baryons, would not rule the soliton models out immediately. That is because there must be vibrational excitations [9, 27] that we have not discussed here, among them an octet similar to that predicted by CQM.

If, however, no other exotics were to be found, how could one get rid of the whole tower of rotational excitations predicted by the soliton models? There has been already some discussion in the literature [27, 28, 18] whether the collective quantization of the rigidly rotating soliton can be applied to the antidecuplet in the first place. Surely, the higher the excitations, the more unreliable is the rigid approximation. Where exactly it breaks down is hard to say, but it cannot even be excluded that the antidecuplet is the first and the last exotic representation for which soliton model predictions still hold.

The confirmed discovery of the Θ^+, together with that of the $\Xi_{\overline{10}}$ if it is also confirmed, usher in a new era of hadron spectroscopy [57]. These developments are already challenging simple versions of the CQM and χSM. Understanding the masses, spin-parities and widths of these exotic baryons and their undiscovered multiplet partners will require a new synthesis of methods in non-perturbative QCD, in which elements of both the CQM and the χSM may play significant rôles.

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